**Collective Excitations (charge)**

We’ll examine density waves and spin waves.

**Particle-Particle Oscillations**

Let’s consider the particle-particle correlation function.



(where n(R,t) is the total site occupation number obtained by summing over spins) Note n(R,t) is number of electrons on the site R, but is not the same as the local electron density n(r,t) = ψ†(r)ψ(r) – can see 2nd quantized file for difference. We’ll opt to work out the corresponding complex time GF,



where δn(**R**,τ) = n(**R**,τ) –n0(**R**,τ), and n0(**R**,τ) just equals ρ0c, a constant, in our case. In terms of c’s and such, we have:



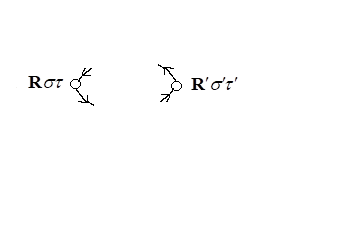
Now how do we calculate the GF? We basically already did the work when calculating the current-current correlation function. Basically I guess, we can say:



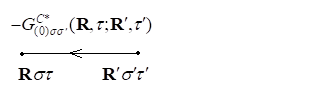
and,

**Real Space Rules**

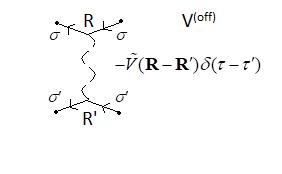
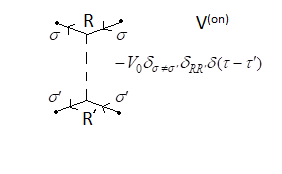
And so we might say our rules are simply, start with external points:



and connect together via the bare GF,

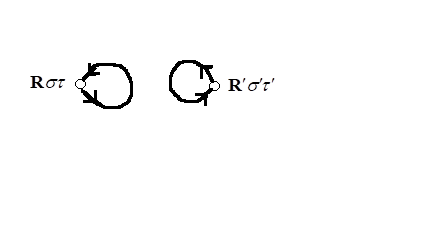


with powers of the interaction vertex:

And then as usual… construct all topologically distinct diagrams/sans vacuum bubbles.

But I think that there is a class of diagrams that should be neglected, and those are the disconnected diagrams. I think all disconnected diagrams can be combined to basically give us a pair of exact GF loops.



And each would be the fully interacting density. But we do not expect the translationally invariant interaction we have to change the local density of particles. They should still be distributed homogeneously throughout the sample. So each should just give us n0. So these guys together will cancel out the -n02 that is hanging around our GF. So then we just need to sum all **fully connected** diagrams. Then we have the usual stuff – each diagram gets factor of one, fermion loops get (-1). Then sum over all internal vertices/times/spins. Seems up to RPA we can treat the spin sum as reducing to factor of two for fermion loops, but with compensatory factor of (1/2) for every V(on) interaction. But don’t forget there is still an overall (-) sgn as well [see the expression above].

**Fourier Space Rules**

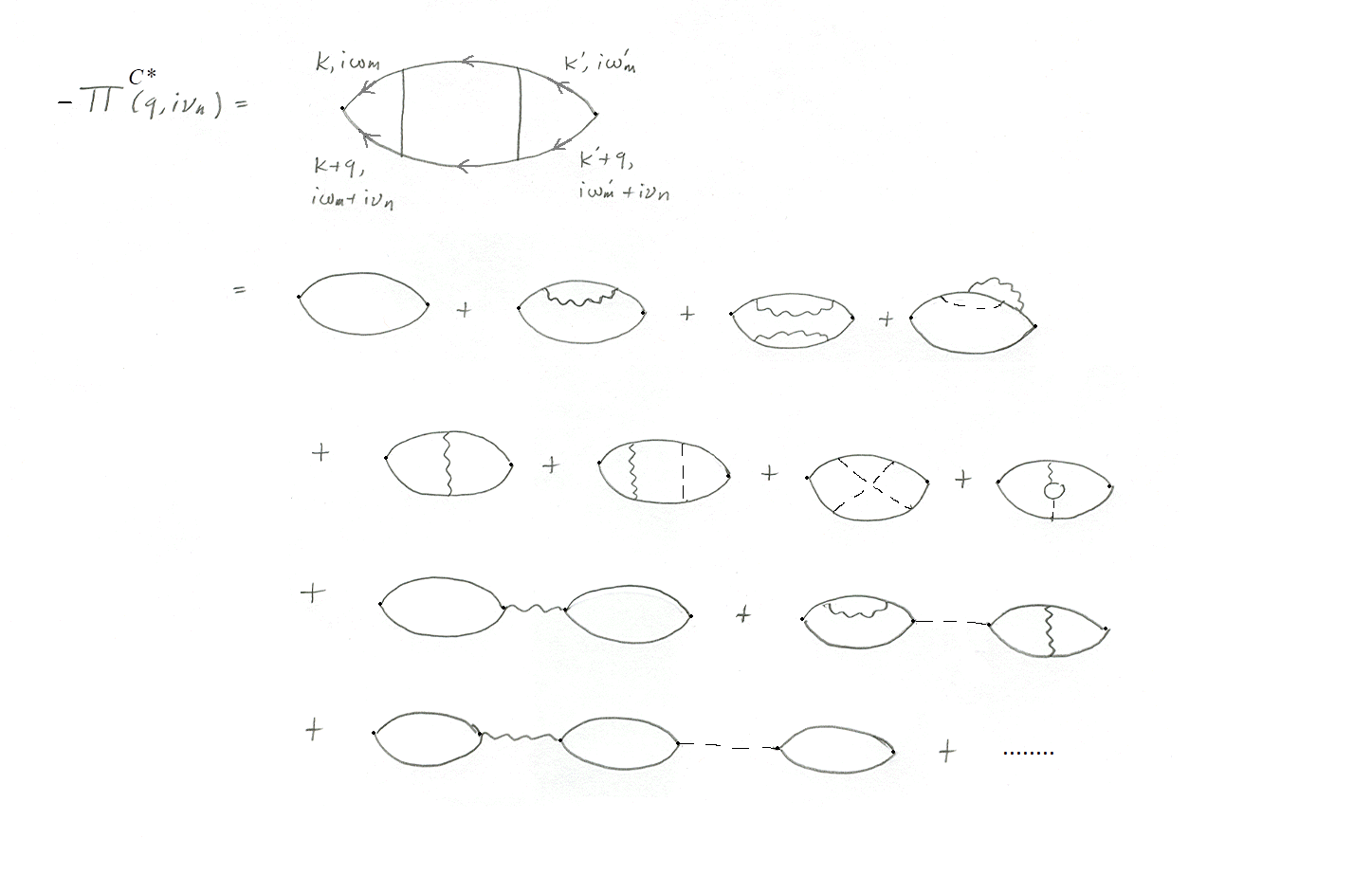
Just to be explicit, what we’re now calculating is the Fourier transform of our correlation function. We can write the correlation function itself as (k sum is only over wavevectors within BZ),



and then taking the Fourier transform,



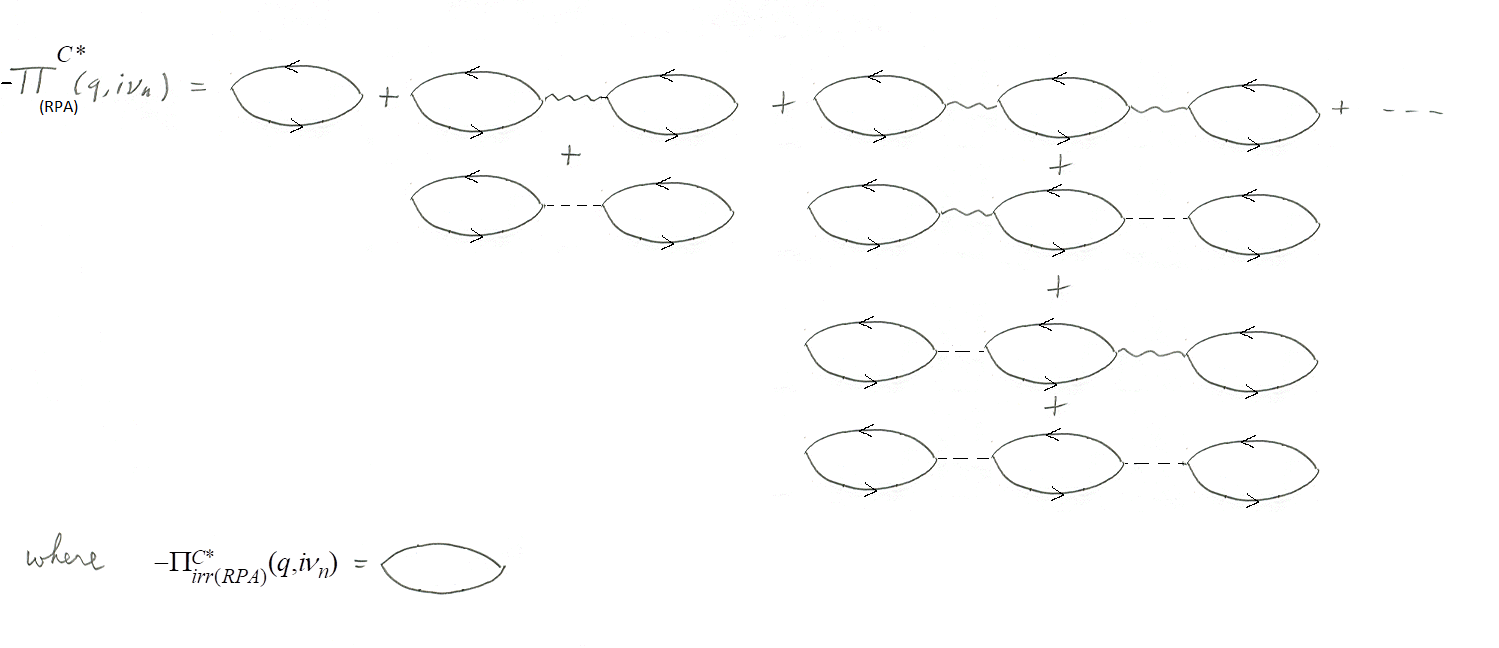
So in any event, re-examine what we did for the current-current correlation function, and what happened when we took the Fourier transform, and we’ll see that we have the following expansion (remember momenta flow against arrows):



Construct all topologically distinct diagrams. Start with ‘dummy’ energy–momentum (k,iωn­) in the top line above the left hand vertex. Impose energy-momentum conservation at each vertex. Add (q,iν­n­) ’boost’ once cross the other vertex. Overall energy-momentum conservation will ensure that energy-momenta ‘below’ the L and R vertices will be (q,iνn­) greater than the momenta above the L and R vertices. Each single unbroken line labeled k,iωn­ represents a factor of G0C\*(k,iωn) as before. And wavy lines give us the interaction of courrse. Fermion loops get (-1). Sum over all unconstrained momenta (including the k and k´) and unconstrained frequencies (including the iωn) with the 1/N and 1/β factor as usual. Quasi-generally, or at least up to RPA, spin sum reduces to factor of two for fermion loops with caveat that V(on) reduces fermion loops by factor of two. But don’t forget the overall [-] too.

**Density-Density Correlation Function Self-Energy (RPA)**

Now we’d like to work out the energy and lifetime of these excitations, which requires extraction of the self-energy. We can depict it diagrammatically as follows, specializing to the RPA limit,



ΠC\*irr(RPA)(q,iνn) functions as the self-energy, and would more generally comprise all diagrams that cannot be separated by cutting through a single interaction line. The corresponding equation would read:



Can motivate the factor of ½ in front of V0. Noting the spin-dependent correlation function must be diagonal in spin ΠC\*irr(RPS)σ1σ2 ~ (1/2)ΠC\*irr(RPA) δσ1σ2, due to fact that G0 is, let’s consider the zeroth order term. Well also, note the (1/2) there is because ΠC\*irr(RPA) technically includes a sum over spins, which results in a factor of 2, which we have to undo, to get the non-summed over spins correlation function.



And now the first order terms,



And for the second order terms, we’d have, just concentrating on the spin sums:



Adding up all the terms, it is evident that we will indeed end up with that expression above. Now continuing,



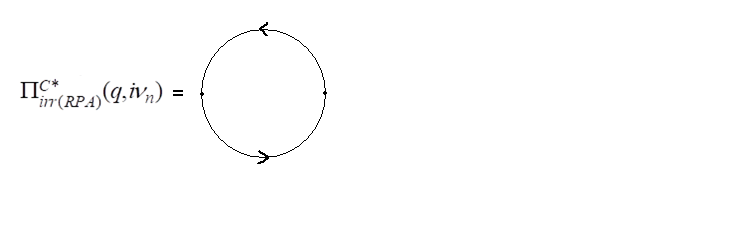
Analytically continuing, we have:



And we’ll also note that this expression would seem to follow automatically from the charge/spin separated Hamiltonian in the Interaction file. I guess that’s one reason why people like to write it that way. From the susceptibilities file, we’ll recognize the denominator as ε(q,ω), but we can treat this as a definition for now, if desired, and so we have:



Note that poles of ΠR will give dispersion spectrum, ω(k). So the zeros of εR(k,ω) give the dispersion relation for these bosonic excitations. Now let’s calculate Πirr(RPA)…



So we have to lowest order:



Since G0C\*(k,iωn) has just one isolated pole, it is straightforward to use the contour integration method to evaluate.



And then,



Analytically continuing, we have:



which is the Lindhardt function from before. Only difference is that the sum is over the BZ, and probably can’t exactly continuum-ize it? Well also, the energy band εk is not free k2/2m, but of tight-binding type.



If we presume a cubic lattice, this would look like:



So Mahan evaluates this in two limits q = ω = 0, and q << ω. In the former case, taking the small q limit, we’ll have:



And in the small q limit, we can approximate the denominator as:



In the zero temperature limit, we just get the density of states at the chemical potential.



and so,



This is just the typical Thomas-Fermi form. In the opposite case ω >> q, Mahan just expands Πirr in a power series in 1/ω. The first term is:



Well how do you handle a sum when the wavevectors extend past the BZ? I guess you just subtract off a reciprocal lattice vector as required, to put them back in the range. In that case, that first sum will just give us 0, since we’ll get the total number of particles each time. Then we’re left with:



Now change summation variable in second sum from k → -k-q. And take advantage of fact that ξ-k = ξk. So then,



where I’m just gonna define a term P(q).



In the small q limit (remember δ≠0),



(assuming linear term vanishes because of symmetry) Let’s just say we’re dealing with a cubic lattice. Then,



So then,



Each of these sums should be the same, sans the q factor. So should be able to write:



where in the penultimate line we recognize that εk = tΣδeik·δ = t[cos(kxδ) + cos(kyδ) + cos(kzδ)]. So in similar spirit, just going to say that:



Now since T < 0 (because t < 0), we can write this as:



So we have all total,



For comparison’s sake, we get nearly the same results for the nearly free model. Refering back to the Free Day/Electrons/Nonequilibrium/Electric Susceptibility file, we found that up to RPA approximation,



So it’s basically the same thing, just a different prefactor P(q). So now back to ΠRRPA(q,ω), in the ω >> q limit,



and so we have:



So we see from the roots here, that we have collective oscillations, and up to this order, they are exact eigenstates since the root is purely real. And these are:



He notes that if we left out the long range interaction, then q → 0, and the roots would go as ω ~ q, which is acoustic. But he says this is obviously not correct. But presuming q ~ 1/q2 as usual, then we’d get ω → ωp in the small q limit, ωp being the plasma frequency. Might parenthetically observe the fact that when we have long range forces, we get oscillations ω → const. as q → 0. But then when we have only short range forces, like when we take out Vq, the oscillations ω → 0 as q → 0.